REPORTS

SIZE EFFECTS ON THE THERMAL CONDUCTIVITY OF THIN METALLIC WIRES: MICROSCALE IMPLICATIONS

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This study examines the influence of radial thickness on the thermal conductivity of thin metallic wires. While size effects on the electrical conductivity of thin wires have been discussed in the literature, research into size effects on thermal conductivity still requires investigation. At such small length scales, the assumption that the reduced electrical conductivity can be simply related to the reduced thermal conductivity through a Wiedemann-Franz relation is subject to question. This study uses the Boltzmann transport equation for electrons to determine the thermal conductivity of a thin wire directly. Electrons are treated as the primary heat carriers in a thin wire with a thermal gradient along the axis. A single-crystal, defect-free, metallic thin wire is considered in the derivation. An expression is presented which accounts for the radial size effects on axial thermal conductivity. The derived thermal conductivity is compared to expressions for the reduced electrical conductivity, and the applicability of the Wiedemann-Franz relation is discussed.

Microscale technology stands on the verge of a device explosion. The electronics industry continues to push rapidly toward the development of a ULSI standard, while MEMS technology is appearing in such diverse fields as military avionics and home appliances. The advent of this micro revolution requires that the physical properties of device materials be properly evaluated. However, as microscale design charges into the submicrometer regime, size effects on the mechanical and electrical properties demand consideration.

Size effects on thermal conductivity will play a decisive role in device modeling in both microelectronics and MEMs technology. In ULSI and VLSI

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circuits, one of the most prominent obstacles to increased speed and calculation power is the interconnects which link different components on the silicon wafer. These metal interconnections are frequently electrically isolated from each other with the use of low-dielectric materials or air. While such a scenario ensures a minimum of electrical interference, the interconnects also become the primary conduits for heat transfer. Size effects on the axial thermal conductivity must be considered in development of these devices. The use of submicrometer interconnections will force the designer to consider lower maximum input currents and the increased possibility for electromigration.

In the construction of MEMS devices, research must focus not only on electrical phenomena but on mechanical considerations as well. Since much of microscale research has been done under a microelectronics paradigm, the advent of MEMS technology forces a reexamination of pertinent physical parameters. Since this technology capitalizes on the use of micrometer-sized moving parts, it must consider a problem faced by all moving systems: the effective transfer of heat. In addition to stress and strain, a clear formulation of the thermal conductivity in microscale regimes will allow MEMS designers to establish more accurate device parameters.

**NOMENCLATURE**

- \( a \) radius of thin wire
- \( A_n \) integrals defined in text
- \( C \) material-dependent constant in definition of relaxation time
- \( e \) electronic charge
- \( E_z \) electric field in the \( z \) direction
- \( f \) electron distribution function
- \( f_0 \) equilibrium distribution for \( f \)
- \( g_0 \) probability density defined in Eq. (4)
- \( G(\varepsilon) \) angular integral defined in Eq. (13)
- \( h \) Planck's constant
- \( I_n \) integrals defined in text
- \( j \) electric current density
- \( k \) thermal conductivity
- \( k_B \) Boltzmann constant
- \( K \) material-dependent parameter in Eq. (35)
- \( l \) function defined in Eq. (7)
- \( m \) electron mass
- \( n \) current exponent in Eq. (35)
- \( p \) fraction of electrons scattered specularly
- \( q(\rho) \) heat flux at given radius
- \( Q \) electromigration activation energy
- \( R_\varepsilon(u) \) ratio of thin wire and bulk electrical conductivities for some \( u \)
- \( t_f \) mean time to failure of an interconnect

- \( T \) temperature
- \( u \) ratio of wire diameter to electron mean free path
- \( v \) magnitude of the velocity vector
- \( \nu \) velocity vector
- \( \rho, \theta, z \) cylindrical coordinates
- \( \beta \) constant defined in Eq. (12)
- \( \varepsilon \) electron's kinetic energy
- \( \lambda \) mean free path
- \( \Lambda_n \) integrals defined in text
- \( \xi \) chemical potential
- \( \sigma \) electrical conductivity
- \( \tau \) relaxation time
- \( \Omega_{1,2} \) integrals defined in Eqs. (31), (32)

**Subscripts**

- \( b \) bulk
- \( b_{total} \) bulk total
- \( e_{bulk} \) bulk electron
- \( e \) electron
- \( ph \) phonon
- \( ph_{bulk} \) bulk phonon
- \( w \) wire
- \( w_{total} \) wire total
The effect of dimension on electrical conductivity has been clearly demonstrated and theoretically evaluated by a number of researchers. Fuchs [2] used the Boltzmann transport equation to model electrical conductivity in thin metallic films. By the same method, Dingle [3] expanded the work to consider the electrical conductivity of thin metallic wires. While a significant amount of research has focused on size effects on electrical conductivity, there have been few theoretical investigations into size effects on thermal conductivity. This is most likely due to the assumption that the Wiedemann-Franz law holds in the microscale regime. Using this assumption, Tien et al. [4] examined the thermal conductivity of thin metallic films and wires and developed approximate expressions for the size dependence. They developed a relationship for the thermal conductivity ratio between thin wire and bulk for situations in which the wire is much larger than the electron mean free path:

$$\frac{k_w}{k_b} = 1 - (1 - p) \frac{3}{4u}$$

(1)

where $k_w$ is the thermal conductivity of the wire, $k_b$ is the bulk thermal conductivity, and $u$ is the ratio of diameter to mean free path, $\lambda$. The factor $p$ represents the portion of electrons which scatter specularly from the crystal surface. In this equation, $p = 0$ when scattering from the surface is assumed to be diffusive.

Tellier and Tosser [5] used geometric arguments to determine a relationship between the reduced thermal conductivity of a thin metal film and a reduction in the electron mean free path due to size effects. More recently, Kumar and Vradis [6] determined an expression for the in-plane thermal conductivity of thin films, following Fuchs’ derivation of the Boltzmann transport equation with an additional term due to the thermal gradient. In their study, they determined that in the first-order approximation, the Wiedemann-Franz relation held for thin metallic films. However, in the report by Kumar and Vradis [6], second-order effects were not discussed.

Despite the equal importance of metal films and wires in modern microelectronics, theoretical research into size effects on the thermal conductivity of thin metal wires has been largely neglected. This work uses the Boltzmann transport formalism and the Fermi-Dirac electron distribution to develop a rigorous expression for the axial thermal conductivity of thin metallic wires. In a similar fashion to Kumar and Vradis [6], this work has built upon Dingle’s [3] work on the electrical conductivity of thin wires. The thin metal wires are assumed to be formed from a single crystal and free of defects, so that the salient relation between size and thermal conductivity can be evaluated.

THEORY

A thin wire with radius $a$ possessing a thermal gradient in the axial direction is considered in the derivation. Thermal conductivity is defined by the heat flow from a thermal gradient in the absence of an electrical current. In order to maintain zero electrical current, an electric field, $E_c$, develops in the axial direction.
of the thin wire to counteract the thermally induced diffusion of electrons. On the order of an electron mean free path, temperature variations are assumed to be small, so that thermalization of the electrons is insured.

The method for derivation follows that of Kumar and Vradis [6]. First the probability density function for electrons in a thermal gradient with size constraints is derived. Then, using the fact that the electrical current density in this case is zero, an expression for $E_z$ is determined. This expression is then used to evaluate the heat flux in the $z$ direction, from which the thermal conductivity can easily be derived.

The Boltzmann transport equation for electrons describes how the electron distribution function is affected by the thermal gradient and boundary constraints of the wire. The transport equation in the present scenario can be expressed as

$$v_{\rho} \frac{\partial f}{\partial \rho} + v_{\theta}^2 \frac{\partial f}{\partial v_{\rho}} - \frac{v_{\rho} v_{\theta}}{\rho} \frac{\partial f}{\partial v_{\theta}} - \frac{eE_z}{m} \frac{\partial f}{\partial v_z} + v_z \frac{\partial f}{\partial T} \frac{dT}{dz} = \frac{f - f_0}{\tau}$$

where $f$ is the electron distribution function, $f_0$ is the equilibrium distribution function, $m$ is the mass of the electron, $e$ is the electron charge, $\tau$ is the relaxation time, $E_z$ is the electric field, and $T$ is the temperature in Kelvin. A velocity vector $v$ is defined with cylindrical coordinates $v_{\rho}, v_{\theta},$ and $v_z$. The first three terms show the size effects of the cylindrical wire, while the fourth and fifth terms represent the movement of the electrons due to the electric field and the diffusion induced by the thermal gradient, respectively.

The deviation of the density function from $f_0$ is assumed to be small, allowing the use of a relaxation time to describe the effect of collisions on returning the distribution to equilibrium. The metal of the wire is considered to be from a single crystal and free of any impurities or grain boundaries. Scattering from the wire surface is taken to be diffusive and elastic. The ambient temperature is assumed to be greater than the Debye temperature of the metal, so that the relaxation time is simply a function of collisions between electrons and phonons. In this region, the relaxation time is a function of the electron’s energy given by

$$\tau = \frac{C \Theta}{T} \varepsilon^{3/2}$$

where $C$ is a constant dependent on the metal in question, $\varepsilon$ is the electron energy, and $\Theta$ is the Debye temperature of the metal [7]. Under equilibrium conditions, the distribution function, $f_0$, for the electrons is given by the Fermi-Dirac distribution:

$$f_0 = 2 \left( \frac{m}{\hbar} \right)^3 g_0$$

$$g_0 = \frac{1}{\exp[(\varepsilon - \xi)/k_B T] + 1}$$
where \( k_B \) is the Boltzmann constant, \( h \) is Planck’s constant, and \( \xi \) is a parameter known as the chemical potential, which is determined by the electron number present.

Assuming that the distribution function is only slightly perturbed from equilibrium, the distribution can be expressed as

\[
f = f_0 + f_1(v, \rho) \tag{5}\]

where \( f_1 \) is a small deviation induced by the thermal gradient. Since \( f \) is of the same order as \( f_0 \), terms containing \( f_1 \) are neglected in comparison to like terms containing \( f_0 \). Applying Eq. (5), the transport equation is found to be

\[
\frac{v_\rho}{\rho} \frac{\partial f_1}{\partial \rho} + \frac{v_\theta}{\rho} \frac{\partial f_1}{\partial \rho} - \frac{v_\rho v_\rho}{\rho} \frac{\partial f_1}{\partial v_\rho} + \frac{f_1}{\tau} = \frac{eE_z}{m} \frac{\partial f_0}{\partial v_z} - \frac{v_z}{\tau} \frac{\partial f_0}{\partial T} \frac{dT}{dz} \tag{6}\]

since \( f_0 \) has no dependence on position. The above equation is just a first-order partial differential equation for \( f_1 \). It is identical to the one solved by Dingle [3], except for an additional constant due to the effect of the thermal gradient. This allows for the use of the general solution obtained by Dingle [3] with a modification due to the additional nonhomogeneous term (see [3] for mathematical details):

\[
f_1 = \frac{eE_z}{m} \frac{\partial f_0}{\partial v_z} - \frac{v_z}{\tau} \frac{\partial f_0}{\partial T} \frac{dT}{dz} \tau \left[ 1 - l(v, \rho) \exp \left( \frac{-\rho v_\rho}{\tau \left( v_\rho^2 + v_\theta^2 \right)} \right) \right] \tag{7}\]

where \( l(v, r) \) is a function of velocity and position determined by the boundary conditions.

In the case of a cylindrical wire, the pertinent boundary conditions are at the wire surface and at the center of the wire, \( \rho = 0 \). Since scattering at the surface is assumed to be diffusive, the electron velocities leaving this region can have no directional dependence, so \( f_1 \) must be zero. At \( \rho = 0 \), the distribution function must be continuous and finite. Evaluating these conditions on \( l(v, r) \) reveals the final form of \( f_1 \):

\[
f_1 = \left( \frac{eE_z}{m} \frac{\partial f_0}{\partial v_z} - \frac{v_z}{\tau} \frac{\partial f_0}{\partial T} \frac{dT}{dz} \right) \tau \left[ 1 - \exp \left( \frac{-\rho v_\rho}{\tau \left( v_\rho^2 + v_\theta^2 \right)} \right) \right] \tag{8}\]

Equation (8) can now be used to calculate the current density \( j \) in the \( z \) direction from the expression

\[
j_z(\rho) = e \iint v_z f_1(v, \rho) \, dv \tag{9}\]
Converting the above integral into spherical coordinates for easier calculation, it assumes the form

\[ j(\rho) = e \int_0^\infty dv \int_0^\pi d\theta \int_0^{2\pi} d\phi \, v^3 \sin \theta \cos \theta \left( \frac{eE_z}{m} \frac{\partial f_0}{\partial v} \cos \theta - v \cos \theta \frac{\partial f_0}{\partial T} \frac{dT}{dz} \right) \]

\[ \times \ \left\{ 1 - \exp \left[ - \left( \frac{\rho \sin \phi + \sqrt{a^2 + \rho^2 \cos^2 \phi}}{\tau \nu \sin \theta} \right) \right] \right\} \tag{10} \]

This integral can now be expressed in terms of electron energy by using the energy dependence of the velocity and the electron mean free path. This gives the integral in the more compact form

\[ j_z(\rho) = \beta \frac{C \Theta e}{T} \left[ \left( eE_z + T \frac{\partial}{\partial T} \frac{dT}{dz} \right) \Lambda_3 + \frac{1}{T} \frac{dT}{dz} \Lambda_4 \right] \tag{11} \]

where

\[ \Lambda_n = - \int_0^\infty d\varepsilon \frac{\partial g_0}{\partial \varepsilon} e^{\varepsilon} G(\varepsilon) \tag{12a} \]

\[ \beta = \frac{4 \sqrt{2} m}{h^3} \tag{12b} \]

and the angular dependence can be expressed as

\[ G(\varepsilon) = \int_0^\pi d\theta \int_0^{2\pi} d\phi \cos^2 \theta \sin \theta \left\{ 1 - \exp \left[ - \left( \frac{\rho \sin \phi + \sqrt{a^2 - \rho^2 \cos^2 \phi}}{\lambda(\varepsilon) \sin \theta} \right) \right] \right\} \tag{13} \]

In the above equation, \( \lambda \) is the electron mean free path, which can be defined in terms of energy as

\[ \lambda = \frac{C \theta}{T} \sqrt{\frac{2}{m}} \varepsilon^2 = \tau \nu \tag{14} \]

The average current density in the \( z \) direction can then be found as

\[ j_z(\rho) = \frac{2}{a^2} \int_0^a \rho j_z(\rho) \, d\rho \tag{15} \]

After performing the required integration, the final form for the average electrical current density along the axis of the wire is obtained. Since there is no
current running through the wire in this case, the average current density can be set equal to zero.

\[ \overline{J_z} = -\beta \frac{C \Theta e}{T} \left( \left\{ eE_z + T \frac{\partial}{\partial T} \left( \frac{\xi}{T} \right) \frac{dT}{dz} \right\} \overline{\lambda_3} + \frac{1}{T} \frac{dT}{dz} \overline{\lambda_4} \right) = 0 \]  

(16)

with

\[ \overline{\lambda_n} = -\int_0^\infty e^{-n} \frac{\partial g_0}{\partial \varepsilon} G(\varepsilon) \, d\varepsilon \]  

(17)

where \( G(\varepsilon) \) is the average of \( G(\varepsilon) \) over the radius of the wire. This allows us to determine the following relation for \( E_z \):

\[ eE_z + T \frac{\partial}{\partial T} \left( \frac{\xi}{T} \right) \frac{dT}{dz} = -\frac{1}{T} \frac{dT}{dz} \overline{\lambda_4} \overline{\lambda_3} \]  

(18)

In a similar fashion, the radial heat flux and average heat flux are given by

\[ q_z(\rho) = \int \int v_z \varepsilon f_1(\varepsilon, \rho) \, dv \]  

(19)

\[ \overline{q_z(\rho)} = \frac{2}{a^2} \int_0^a \rho q_z(\rho) \, d\rho \]

Using Eq. (19), the average heat flux in the \( z \) direction can be written as

\[ \overline{q_z(\rho)} = \beta \frac{C \Theta}{T^2} \frac{dT}{dz} \left( \frac{\overline{\lambda_3 \overline{\lambda_5} - \overline{\lambda_4 \overline{\lambda_4}}}}{\overline{\lambda_3}} \right) \]  

(20)

which yields for the thermal conductivity in the axial direction,

\[ k_w = \beta \frac{C \Theta}{T^2} \left( \frac{\overline{\lambda_3 \overline{\lambda_5} - \overline{\lambda_4 \overline{\lambda_4}}}}{\overline{\lambda_3}} \right) \]  

(21)

A similar derivation can be performed to determine the bulk thermal conductivity for large wires, where the effects of the metal surface are negligible. The deviation from the equilibrium distribution is now simply a function of electron velocity. The angular term for the bulk case, \( G_b \), has no dependence on energy and can be taken out of the integral. The calculations yield the following expression for the bulk thermal conductivity:

\[ k_b = \beta \frac{C \Theta G_b}{T^2} \left( \frac{I_3 I_5 - I_4 I_4}{I_3} \right) \]  

(22)
where

\[ I_n = -\int_0^\infty d\varepsilon \frac{\partial g_0}{\partial \varepsilon} \varepsilon^n \]  \hspace{1cm} (23a)

\[ G_b = \frac{4\pi}{3} \]  \hspace{1cm} (23b)

We can now examine the ratio of the thermal conductivities to determine how it compares to previous approximations:

\[ \frac{k_w}{k_b} = \frac{1}{G_b} \frac{\prod_{3} A_3 \prod_{4} A_4}{\prod_{3} I_3 I_5 - I_4 I_4} I_3 \]  \hspace{1cm} (24)

absorbing the \( G_b \) term into \( \prod_n A_n \), the equation can be rewritten as

\[ \frac{k_w}{k_b} = \frac{A_3 A_5 - A_4 A_4}{A_3 I_3 I_5 - I_4 I_4} \frac{I_3}{I_3 I_5 - I_4 I_4} \]  \hspace{1cm} (25)

where

\[ A_n = -\int_0^\infty d\varepsilon \frac{\partial g_0}{\partial \varepsilon} \varepsilon^n R_g (\varepsilon) \]  \hspace{1cm} (26)

\( R_g (\varepsilon) \) is the ratio of the angular terms \( \overline{G(\varepsilon)} \) and \( \overline{G_b} \). Upon examination, it can be shown that \( R_g (\varepsilon) \) is identical to the ratio of electrical conductivities between a thin wire and bulk determined by Dingle [3]. Dingle was able to simplify this ratio to the following form:

\[ \frac{\sigma_w}{\sigma_b} = R_g (\varepsilon) = 1 - \frac{12}{\pi} \int_0^1 dx \sqrt{1 - x^2} \int_0^{\pi/2} d\theta \sin \theta \cos^2 \theta \exp \left( \frac{-ux}{\sin \theta} \right) \]  \hspace{1cm} (27)

where \( u \) is the ratio of the diameter of the wire to the electron mean free path.

In this present form, the ratio of thermal conductivities is not very illuminating and also computationally difficult to calculate. To express this ratio in more physically relevant terms, the integrals over energy can be approximated using the following asymptotic expansions [7]:

\[ A_n = \xi^n R_g (\xi) + \frac{\pi^2}{6} (k_BT)^2 \frac{d^2}{d\xi^2} \frac{\xi^n R_g (\xi)}{d\xi^2} + \frac{7\pi^4}{360} (k_BT)^4 \frac{d^4}{d\xi^4} \frac{\xi^n R_g (\xi)}{d\xi^4} + \ldots \]  \hspace{1cm} (28)

\[ I_n = \xi^n + \frac{\pi^2}{6} (k_BT)^2 \frac{d^2}{d\xi^2} \frac{\xi^n}{d\xi^2} + \frac{7\pi^4}{360} (k_BT)^4 \frac{d^4}{d\xi^4} \frac{\xi^n}{d\xi^4} + \ldots \]  \hspace{1cm} (29)
Keeping second-order terms in the expansion, the thermal conductivity can be expressed as

\[
\frac{k_w}{k_b} \approx R_g(\xi) \left\{ 1 - \frac{3u\Omega_1 + 2u^2\Omega_2}{3u\Omega_1 + 2u^2\Omega_2 + 6R_g(\xi)\left(1 + \frac{\xi^2}{(\pi k_b T)^2}\right)} \right\}
\]  

(30)

where

\[
\Omega_1 = \frac{12}{\pi} \int_0^1 dx \int_0^{\pi/2} d\theta \frac{x^2 \cos^2 \theta \exp \left(\frac{-ux}{\sin \theta}\right) - \sin \theta}{x \sqrt{1 - x^2}}
\]

(31)

\[
\Omega_2 = \frac{12}{\pi} \int_0^1 dx \int_0^{\pi/2} d\theta \frac{x^2 \sqrt{1 - x^2} \cos^2 \theta \exp \left(\frac{-ux}{\sin \theta}\right) - \sin \theta}{x \sqrt{1 - x^2}}
\]

(32)

We can simply express the ratio of thermal conductivities as \( R_g(\xi) \), which is identical to the expression for the ratio of electrical conductivities that Dingle [3] found for a thin metallic wire. As a result, in the first-order approximation, the Wiedemann-Franz relation holds in the microscale regime and the ratio of thermal conductivities is equivalent to the ratio of electrical conductivities:

\[
\frac{k_w}{k_b} \approx R_g(\xi) = \frac{\sigma_w}{\sigma_b}
\]

(33)

In the limit where \( u \gg 1 \), the exponential term in the integral can be expanded and the following approximation for the thermal conductivity is found:

\[
\frac{k_w}{k_b} \approx 1 - \frac{3}{4u^2} + \frac{3}{8u^3} + \ldots
\]

(34)

This expression is identical to that found by Dingle [3] for the ratio of electrical conductivities for large-diameter wires. The first two terms in this expansion are identical to Tien et al.’s [4] approximation for the thermal conductivity ratio of thin wires.

**RESULTS**

The above expressions for thermal conductivity were evaluated using Mathematica on a Unix workstation. Examining the first-order approximation, it was found that size effects become important when the diameter of the wire becomes roughly 10 times that of the electron mean free path (Figure 1).

For metals commonly used in microelectronics, such as Al, Ag, and Au, the thermal conductivity is reduced in wires with diameters of roughly 0.5 \( \mu \text{m} \) or less. When the diameter of the thin wire is of the same order as the electron mean free
Figure 1. Comparison of thermal conductivity ratios.

path, the thermal conductivity drops by roughly half of the bulk thermal conductivity. In this length regime, the diffusive nature of the wire surface plays a significant role in impeding the transfer of heat in the axial direction. The expression developed by Tien et al. [4] is also shown for comparison. This approximation was found to be reliable in the region \( u \gg 10 \), where \( u \) is the ratio of diameter to electron mean free path.

Second-order effects were also considered to determine the regimes where deviations from the Wiedemann-Franz relation occur. The thermal conductivity ratio including second-order terms for Ag at 300 K is shown in Figure 2. As can be seen here, there is no discernible difference between the first-order expression of the thermal conductivity and the form using second-order terms. Examining individual values of each expression, the deviation was found to be less than 0.01%. The influence of the second-order terms will become more significant as the temperature of the sample increases. However, since \( \partial g_0/\partial \varepsilon \) possesses a very strong delta function character around \( \xi \), the required temperature increase for the effects to be noticeable is well outside the range of normal microscale applications.

The specific dependence on size for thin wires of a variety of metals is shown in Figure 3. The nature of the thermal conductivity ratio is strongly dependent on the electron mean free path. Gold and copper possess mean free paths of 41.7 and 42.4 nm, respectively, while silver’s electron mean free path is 55.6 nm. This difference in electron mean free path leads to silver experiencing a greater size effect on the thermal conductivity ratio.

In the regions where the radius of the wire is comparable to the mean free path, the reduced thermal conductivity will cause metallic interconnects to experience higher average temperatures. The mean time to failure of an interconnect can
be expressed by Black’s formula:

\[ t_f = K j^{-n} \exp \left( \frac{Q}{k_B T} \right) \]  

(35)

where \( K \) depends on structural properties of the thin wire, \( j \) is the current density, \( n \) is the current exponent, and \( Q \) is the activation energy for electromigration induced failure [8]. Equation (35) indicates that as \( T \) increases, the mean time to
failure drops rapidly. Thus, size effects on the thermal conductivity must be accounted for in the thermal design of microelectronic components.

Since this model assumes that electrons are the only carriers of heat, further analysis will be necessary to evaluate the thermal conductivity of metals where phonons play an appreciable role in heat transfer, e.g., bismuth. A rough estimate of the total thermal conductivity for a thin metal wire can be obtained by incorporating the effective phonon thermal conductivity for wires developed by Walkauskas et al. [9]. If diffusive scattering is assumed and an isotropic phonon mean free path is considered, an expression can be developed for situations where the diameter of the wire is much greater than the phonon and electron mean free paths:

$$k_{w\_total} = k_{b\_total} - k_{e\_bulk} \left( \frac{3}{4u_e} - \frac{3}{8u_e^3} \right) - k_{ph\_bulk} \left( \frac{3}{2u_{ph}} - \frac{3}{4u_{ph}^3} \right)$$

(36)

where $u_{ph}$ is the ratio of the wire diameter to the phonon mean free path and $u_e$ is the same ratio for electrons.

**CONCLUSIONS**

The Boltzmann transport formalism was used to evaluate the effect of size on the thermal conductivity of thin metallic wires. In situations where the diameter of the thin wire is 10 times the electron mean free path or less, the thermal conductivity of the wire was shown to be appreciably different from bulk values. Since the electron mean free path for metals is normally of the order of 10 nm, a reduced thermal conductivity in submicrometer interconnects and devices must be considered. In the first-order approximation, the electrical and thermal conductivity ratios were found to be identical. In most practical microscale enterprises, the Wiedemann-Franz relation can be reliably used to interpret thermal conductivity. This study also examined the influence of second-order effects on the total thermal conductivity. These effects were found to be negligible in most practical situations.

An estimate for the size effect for the total thermal conductivity for a metal wire was developed for the case of large wire diameters. Consideration of effects of different phonon modes in combination with the electronic thermal conductivity could provide a strong model for the overall thermal conductivity.

The thermal conductivity was evaluated for the case of completely diffusive scattering, where $p = 0$. Due to a strong correlation between $p$ and the actual surface roughness of a sample, the specularity constant can have a large effect on the predicted thermal conductivity. Future work will focus on developing a relationship between the specular coefficient and surface roughness through the use of Monte Carlo simulations and experimental measurement of the electrical conductivity of thin wires. The effect of impurities and grain boundaries will also be incorporated into the present model to develop a more robust expression.

Size effects on thermal conductivity and other mechanical parameters cannot be overlooked in the development of microscale devices. As industry continues to push toward miniaturization, factors which were once important only at cryogenic
temperatures (such as the electronic mean free path and the surface roughness of interfaces) must now be considered even at room temperature.

REFERENCES