The Steady-State Modeling and Optimization of a Refrigeration System for High Heat Flux Removal

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Abstract

Steady-state modeling and optimization of a refrigeration system for high heat flux removal, such as electronics cooling, is studied. The refrigeration cycle proposed consists of multiple evaporators, liquid accumulator, compressor, condenser and expansion valves. To obtain more efficient heat transfer and higher critical heat flux (CHF), the evaporators operate with two-phase flow only. This unique operating condition necessitates the inclusion of a liquid accumulator with integrated heater for the safe operation of the compressor. Due to the projected incorporation of microchannels into the system to enhance the heat transfer in heat sinks, the momentum balance equation, rarely seen in previous vapor compression cycle heat exchangers modeling efforts, is utilized in addition to the mass and en-

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ergy balance equations to capture the expected significant microchannel pressure drop witnessed in previous experimental investigations. Using the steady-state model developed, a parametric study is performed to study the effect of various external inputs on the system performance. The Pareto optimization is applied to find the optimal system operating conditions for given heat loads such that the system coefficient of performance (COP) is optimized while satisfying the CHF and other system operation constraints. Initial validation efforts show the good agreement between the experimental data and model predictions.

Key words:  High heat flux, vapor compression cycle, CHF, COP, Pareto optimization

Nomenclature

\( c_p \) constant pressure specific heat (kJ/kg·K)

\( c_v \) constant volume specific heat (kJ/kg·K)

\( h \) enthalpy (kJ/kg)

\( \dot{m} \) mass flow rate (kg/s)

\( p \) pressure (kPa)

\( q'' \) heat flux (kW/m²)

\( q_a \) heat rate supplied to the accumulator (W)

\( q_c \) heat rate rejected by the condenser (W)

\( q_e \) evaporator heat load (W)
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<th>Symbol</th>
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<tr>
<td>$z$</td>
<td>flow length (m)</td>
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<tr>
<td>$A$</td>
<td>heat exchanger cross sectional area (m$^2$)</td>
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<tr>
<td>$A_v$</td>
<td>valve opening (%)</td>
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<tr>
<td>$K_v$</td>
<td>valve orifice coefficient ($m^2$)</td>
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<tr>
<td>$L_c$</td>
<td>length of condenser (m)</td>
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<tr>
<td>$M$</td>
<td>refrigerant charge (kg)</td>
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<tr>
<td>$S$</td>
<td>heat exchanger surface area (m$^2$)</td>
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<td>$T$</td>
<td>temperature (°C)</td>
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<td>$U$</td>
<td>overall heat transfer coefficient (W/m$^2$·K)</td>
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<tr>
<td>$V_d$</td>
<td>compressor displacement volume (m$^3$/rpm)</td>
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<td>$W$</td>
<td>power (W)</td>
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<tr>
<td>$\eta_s$</td>
<td>isentropic efficiency</td>
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<td>volumetric efficiency</td>
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<tr>
<td>$\omega$</td>
<td>compressor speed (rpm)</td>
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<td>$\rho$</td>
<td>density (kg/m$^3$)</td>
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<td>$a$</td>
<td>accumulator</td>
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<td>$act$</td>
<td>active charge</td>
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1. INTRODUCTION

Since its invention in the early 19th century, refrigeration systems have been used in numerous applications ranging from food storage to metal processing industries. In the most widely used applications such as refrigerators, automobiles, and residential and public buildings, the enclosure temperatures are regulated. In recent years, however, there has been an increasing demand for refrigeration systems targeted at high heat flux removal. Spurred by the ever-increasing packaging density and, hence, power density in electronic systems, heat flux dissipated
from semiconductor devices could reach as high as 0.1\(kW/cm^2\) \(\sim\) 10\(kW/cm^2\) [1, 2, 3, 4, 5, 6, 7] for such diverse applications as active radars, all-electric ships, high power light emitting diodes (LED), hybrid electric vehicles (HEV), and semiconductor lasers. More than ever, high heat flux dissipation has become the bottleneck toward compact higher performance electronic systems, and heat flux management is becoming more challenging to maintain product reliability and desired life time. Thermal challenges have spawned the so-called thermal-aware design in the electronics industry, and a number of innovative cooling techniques have been proposed which include refrigeration systems with microchannel heat sinks [8], thermoelectric coolers [9], and liquid impingement cooling [10, 11, 12]. Among these new cooling techniques, refrigeration systems with microchannel heat sinks have been shown to be particularly promising. Mudawar [8], for example, has shown that a heat flux of 840\(W/cm^2\) can be successfully removed without triggering the CHF condition by using a refrigeration system with microchannels.

Tremendous efforts have been devoted to the modeling of conventional vapor compression cycles. In dynamic modeling, the moving boundary approach is used to model the heat exchangers in the early work of Wedekind [13]. This approach is later adopted in the work of Grald [14], He [15] and the recent development of the vapor compression cycles simulation tool “Thermosys” [16, 17] by Rasmussen. The finite volume approach is used to model heat exchangers in [18]. In steady-state modeling, which is more related to the present work, Richardson et al. [19, 20] develop the simulation tool “VapCyc” for the steady-state analysis of vapor compression cycles that allows the optimization of refrigerant charge and design for different components configurations. A correlation is used to capture the pressure drop in the heat exchangers; therefore the momentum balance
equation is not included. Lin [21] briefly introduces the steady-state optimization of an air-conditioning system based on COP. The optimal operating conditions are obtained by adjusting the compressor speed and the expansion valve opening through constrained nonlinear optimization. Corberán develops a computer code “ART” [22] for the steady-state design of refrigeration and air-conditioning equipments, and the momentum balance equation is used in the modeling of the heat exchangers.

In recent years, there has been an increasing interest in using vapor compression cycles for high heat flux cooling [8, 23, 24, 25, 26]. The system designers, however, are not armed with the appropriate design or optimization tools to help them select the components for each particular high heat flux cooling system or determine the ideal operating conditions for given heat loads. Targeting conventional air-conditioning systems, the modeling and optimization work discussed above either neglects the momentum balance equation in the modeling of heat exchangers [16, 17, 19, 20] or is restricted to a superheated condition at the exit of the evaporator [18, 21, 22]. These two fundamental deficiencies limit the application of the previous modeling and optimization work in the high heat flux removal scenarios because the significant pressure drop in the present microchannels will not be captured, and a superheated evaporator exit will most probably lead to unacceptably high device temperatures.

Aiming to bridge this gap, the current work introduces a general modeling and optimization framework for high heat flux electronics cooling. A refrigeration system structure for high heat flux removal is proposed, and the corresponding steady-state system modeling and optimization are investigated. The CHF consideration is a fundamental difference between the present refrigeration sys-
tem targeted at high heat flux removal and the conventional refrigeration systems
designed for temperature regulation. In our case, the evaporator is designed to
operate with two-phase flow only for more efficient heat transfer and higher CHF,
and, thus, a liquid accumulator with an integrated heater is placed downstream of
the evaporator to ensure the safe operation of the compressor. Compared with the
conventional refrigeration systems, which are designed to operate within a small
range around a fixed operating point, the refrigeration system proposed here for
high heat flux removal needs to cover a wide range of operating conditions, as
the heat generating device could operate anywhere from the standby mode to full
load mode and result in various levels of heat fluxes. The varying active charge of
the refrigeration system corresponding to the wide range of operating conditions
is accommodated by the liquid level change in the accumulator, and the active
charge here is defined as the total refrigerant holdup in the heat exchangers and
piping.

The rest of this paper is structured as follows. Section 2 presents the refriger-
eration system structure for high heat flux removal and the detailed modeling of
individual cycle components. In section 3, the steady-state cycle simulation
setup is described and a parametric study is performed to investigate the effect of
various external inputs on system performance. In section 4, the Pareto optimiza-
tion approach is applied to find the optimal steady-state operating conditions to
remove the given heat fluxes, and the optimization results are interpreted in de-
tails. The experimental testbed and initial model validation results are discussed in
section 5. Finally, section 6 concludes the present work and provides suggestions
for future work.

It is expected that the system structure and design methodology proposed in
this paper are applicable to high heat flux removal applications of various scales, and that the modeling, simulation, and optimization framework will aid throughout the system design process from component sizing to subsequent steady-state operation optimization. By varying the system setup, the same approach is also readily applicable to the design of traditional refrigeration cycles for temperature regulation. Table 1 compares the present study and related work, highlighting the CHF and momentum balance considerations in the present study which are essential in high heat flux removal applications using microchannels.

2. MODELING OF REFRIGERATION SYSTEM FOR HIGH HEAT FLUX REMOVAL

The present refrigeration system structure for high heat flux removal is depicted in Fig. 1. The five components of the system are evaporator, liquid accumulator with integrated heater, variable speed compressor, condenser, and electronic expansion valve (EEV). For simplification, pressure drop and refrigerant storage in the piping are not considered.

The fundamental feature that differentiates the present refrigeration system in Fig. 1 from other vapor compression cycles proposed for high heat flux removal is that the evaporator operates with two-phase flow only. In the high heat flux scenario, the high heat transfer coefficient associated with two-phase flow helps keep the surface temperature of heat generating device low. In comparison, other high heat flux electronics cooling work [23, 24, 26] has superheated vapor at the evaporator exit. The low heat transfer coefficient of superheated vapor, together with an imposed high heat flux, will inevitably lead to unacceptably high device temperatures and, thus, the heat generating device has to be placed in contact with the two-phase flow region of the evaporator only. In addition to the device
placement issue, the big difference in heat transfer coefficients between the two-phase and superheated flow regions will also result in less uniformity in the device temperature distribution, which induces more thermally-induced strain.

For the protection of the compressor, a liquid accumulator is included downstream of the evaporator. Heat from the integrated accumulator heater fully evaporates the two-phase flow coming out of the evaporator, and, thus, only saturated vapor enters the compressor at steady state. Through its liquid level change, the accumulator also has the important role of cycle active refrigerant charge regulation to accommodate a wide range of operating conditions.

The external inputs of the refrigeration system are heat load of the evaporator $q_e$, heat provided to the accumulator from the integrated heater $q_a$, compressor speed $\omega$, temperature of the second fluid to which the heat is rejected $T_{cool}$, and EEV percentage opening $A_{ev}$. The steady state of the system is fully represented by the pressure and enthalpy at the exit of each cycle component, together with the cycle mass flow rate. As shown in the detailed component modeling below, each cycle component is described by two equations except the accumulator. The accumulator, in addition to the two equations representing energy balance and pressure, has a third equation stating that the accumulator exit is saturated vapor in steady state. With this third accumulator equation, the loop of interconnected cycle components is closed and cycle steady state can be solved. In comparison, the conventional vapor compression cycle without accumulator uses the cycle refrigerant charge conservation equation to close the loop.

2.1. Compressor

The compressor mass flow rate is given by:

$$\dot{m} = \omega V_d \rho_v \eta_c \tag{1}$$
where $\omega$, $V_d$, $\rho$, and $\eta_v$ are compressor speed (rpm), displacement volume, inlet refrigerant density, and volumetric efficiency, respectively. The volumetric efficiency $\eta_v$ can be approximated by [18]:

$$
\eta_v = 1 + c_r - d_r (p_o/p_i)^{c_v/c_p}
$$  \hspace{1cm} (2)

where $c_r$ and $d_r$ are coefficients to be identified from the actual compressor, $p_o$ and $p_i$ are the compressor outlet and inlet pressures, and $c_v$ and $c_p$ are constant volume and constant pressure specific heats evaluated at the compressor inlet condition.

The compressor exit enthalpy is given by:

$$
h_o = h_i + (h_{is} - h_i)/\eta_s
$$  \hspace{1cm} (3)

where $h_o$, $h_i$, and $h_{is}$ are compressor outlet, inlet, and isentropic enthalpy, respectively. The compressor isentropic efficiency $\eta_s$ varies with operating conditions and can be described with the following polynomial approximation [27]:

$$
\eta_s = c_0 + c_1 \omega + c_2 \omega^2 + c_3 (p_o/p_i) + c_4 (p_o/p_i)^2
$$  \hspace{1cm} (4)

where the coefficients $c_0$, $c_1$, $c_2$, $c_3$ and $c_4$ can be identified from the actual compressor.

Assuming the compression process is adiabatic, the work done by the compressor $W_m$ can be calculated using the mass flow rate, and inlet and outlet enthalpies:

$$
W_m = \dot{m}(h_o - h_i)
$$  \hspace{1cm} (5)

2.2. Electronic Expansion Valve (EEV)

The cross sectional area of the EEV can be electronically controlled and can be translated into a percentage opening denoted by $A_v$ (ranging from 0 to 100). It is
assumed that the expansion process is an adiabatic constant enthalpy process with $h_o = h_i$, where $h_o$ and $h_i$ are the expansion valve’s outlet and inlet enthalpies.

The pressure drop across the expansion valve is:

$$\Delta p = \frac{\dot{m}^2}{K_v A_{h}^2 \rho_i}$$

(6)

where $K_v$ is the expansion valve orifice coefficient obtained from the manufacturer (or through experiments), and $\rho_i$ is refrigerant density at the inlet of the expansion valve.

2.3. Liquid Accumulator

Both the inlet and the outlet of the liquid accumulator are located at its top. At steady state, only saturated vapor leaves the accumulator and thus:

$$p_o = p_i$$

(7)

$$h_o = h_{sat\text{--}vap}(p_o)$$

(8)

where $p_o$ and $p_i$ are the outlet and inlet pressures of the accumulator, $h_o$ is the accumulator exit enthalpy, and $h_{sat\text{--}vap}(p_o)$ is the saturated vapor enthalpy at pressure $p_o$.

At steady state, the accumulator level does not change, and the incoming two-phase flow at the accumulator inlet is fully evaporated into saturated vapor leaving the accumulator by $q_a$, heat supplied to the accumulator heater. Assuming that the accumulator is insulated, from energy balance we have:

$$q_a = \dot{m}(h_o - h_i)$$

(9)

where $h_i$ is the refrigerant enthalpy at the accumulator inlet. As explained earlier, this energy balance equation plays the important role of closing the loop in later cycle simulations.
With a known total refrigerant charge of the vapor compression cycle $M_{\text{total}}$, and the active charge $M_{\text{act}}$ calculated from the exchangers and piping, the total refrigerant holdup in the liquid accumulator $M_a$ is $M_a = M_{\text{total}} - M_{\text{act}}$. Once the geometry of the liquid accumulator is given, $M_a$ can be easily converted to the liquid level of the accumulator.

2.4. Heat Exchangers

A one-dimensional homogeneous model is used to characterize both the evaporator and condenser. The steady-state behavior of the refrigerant inside the heat exchangers can be described by the mass, energy, and momentum balance equations:

\[ \frac{\partial \dot{m}}{\partial z} = 0 \]  
\[ \frac{\partial (\dot{m}h)}{\partial z} + q'' S = 0 \]  
\[ \frac{1}{A} \frac{\partial (\dot{m}^2/\rho)}{\partial z} + A \frac{\partial p}{\partial z} + F_f = 0 \]

where $z$ stands for the location measured from the inlet of the heat exchangers, $A$ is the flow cross-sectional area, $S$ is the perimeter of the cross section area, $q''$ is the incoming heat flux (negative in the case of condenser), and $F_f$ is the term introduced to describe the friction inside the channels of the heat exchangers.

At steady state, the energy and momentum balance equations can be integrated along the heat exchanger length to obtain the following two algebraic equations:

\[ h_o = h_i + \frac{q}{\dot{m}} \]  
\[ \Delta p \triangleq p_i - p_o = (\frac{\dot{m}}{A})^2 \left( \frac{1}{\rho_o} - \frac{1}{\rho_i} \right) + \Delta p_f \]

where $h_i$, $\rho_i$, and $p_i$ are the enthalpy, density and pressure at the heat exchanger inlet, while $h_o$, $\rho_o$, and $p_o$ are the enthalpy, density and pressure at the heat ex-
changer outlet. The pressure drop across the heat exchanger $\Delta p$ is the sum of the acceleration pressure drop $\left(\frac{\dot{m}}{A}\right)^2\left(\frac{1}{\rho_o} - \frac{1}{\rho_i}\right)$ and the frictional pressure drop $\Delta p_f$ calculated using the Martinelli correlation [28].

The rate of heat transferred by the heat exchanger $q$ is calculated according to the different boundary conditions in the evaporator and condenser, and established correlations are used to predict the heat transfer coefficients in different flow regimes. In the single-phase region of the heat exchangers, the Petukhov and Popov correlation [29], Gnielinski correlation [30], and Kandlikar correlation [31] are used for highly turbulent, turbulent, laminar and transitional flow, respectively. In the two-phase region, the Kandlikar correlation [31] is used for boiling, and the correlation by Shah [32] is used for condensation.

2.4.1. Evaporator

A uniform heat flux $q''$ is applied to the evaporator, and the heat transfer rate into the evaporator $q_e$ is:

$$q_e = q''S_e$$  \hspace{1cm} (15)

where $S_e$ is the evaporator surface area.

The evaporator wall temperature $T_{wall}$ at any specific location is:

$$T_{wall} = T_r + \frac{q''}{U}$$  \hspace{1cm} (16)

where $T_r$ and $U$ [29, 30, 31] are the locally evaluated refrigerant temperature and overall heat transfer coefficient, respectively.

2.4.2. Condenser

Depending on the operation conditions, the condenser can operate with two zones (superheated and two-phase) or three zones (superheated, two-phase, and
subcooled). It is thus described by a multi-zone moving boundary model in which both the number of zones in the condenser and the length of each zone may vary.

In each zone of the condenser, the integrated energy and momentum balance equations are:

\[ h_{o,k} = h_{i,k} - \frac{q_{c,k}}{\dot{m}} \]  

(17)

\[ \Delta p_{c,k} \triangleq p_{i,k} - p_{o,k} \]

\[ = \left(\frac{\dot{m}}{A_c}\right)^2 \left(\frac{1}{\rho_{o,k}} - \frac{1}{\rho_{i,k}}\right) + \Delta p_{f,k} \]  

(18)

where \( A_c \) is the cross sectional area of the condenser, and \((h_{i,k}, \rho_{i,k}, p_{i,k})\) and \((h_{o,k}, \rho_{o,k}, p_{o,k})\) are the enthalpy, density and pressure evaluated at the inlet and outlet of the \( k^{th} \) zone, respectively. The pressure drop across the \( k^{th} \) zone \( \Delta p_{c,k} \) is the sum of the acceleration pressure drop \( \left(\frac{\dot{m}}{A_c}\right)^2 \left(\frac{1}{\rho_{o,k}} - \frac{1}{\rho_{i,k}}\right) \) and the frictional pressure drop \( \Delta p_{f,k} \).

The rate of heat rejected in the \( k^{th} \) zone to the second fluid \( q_{c,k} \) is:

\[ q_{c,k} = U_k S_k \left[ \frac{(T_{o,k} - T_{cool}) - (T_{i,k} - T_{cool})}{\ln\left[\frac{T_{o,k} - T_{cool}}{T_{i,k} - T_{cool}}\right]} \right] \]  

(19)

where \( U_k \) [29, 30, 32], \( S_k \), \( T_{cool} \), \( T_{i,k} \), and \( T_{o,k} \) are the overall heat transfer coefficient, surface area, second fluid temperature, and inlet and outlet refrigerant temperatures of the \( k^{th} \) zone.

The rate of total heat rejected by the condenser \( q_c \) is:

\[ q_c = \sum_{k=1}^{N} q_{c,k} \]  

(20)

where \( N \) (2 or 3) is the total number of zones existing in the condenser. The length of each zone \( L_{c,k} \) should be greater than zero and add to the total length of
the condenser $L_c$:

\[
L_c = \sum_{k=1}^{N} L_{c,k}. \tag{21}
\]

Note that at the boundary between the superheated and two-phase regions $h = h_{\text{satvap}}(p)$, in which $h_{\text{satvap}}(p)$ stands for the saturated vapor enthalpy at pressure $p$. Similarly, $h = h_{\text{satliq}}(p)$ holds true at the boundary between the two-phase and subcooled regions, in which $h_{\text{satliq}}(p)$ stands for the saturated liquid enthalpy at pressure $p$.

### 3. SYSTEM SIMULATION SETUP AND PARAMETRIC STUDY

#### 3.1. Cycle Setup

In an effort to reflect the testbed built at RPI for high heat flux removal, the refrigeration system simulated has exactly the same structure shown in Fig. 1, except that the single evaporator in Fig. 1 is replaced by three identical evaporators in parallel configuration. Each evaporator is equipped with an EEV of the same model to regulate its mass flow rate. R134a is chosen to be the cycle refrigerant because of its wide use, but other refrigerants can also be easily used in the simulation if their thermophysical property tables are available. On the condenser side, water is chosen to be the second fluid, and its temperature is denoted as $T_{\text{cool}}$.

The three evaporators are configured to have the same operating conditions; thus, $q_{e,1} = q_{e,2} = q_{e,3} = q_e$, $A_{v,1} = A_{v,2} = A_{v,3} = A_v$, and $\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}/3$. Cycle simulation results with nonequal evaporator heat loads have also been obtained but are not included here because of space limitations.

#### 3.2. System Performance Indices

For the present refrigeration system targeted at high heat flux removal, CHF is the safety index and COP is the efficiency index.
3.2.1. Coefficient of Performance

For refrigeration cycles, COP can be defined as the ratio of the heat transfer rate received by the system undergoing the cycle from the cold body, \( q \), to the net power into the system to accomplish this effect, \( W_{\text{cycle}} \) [33].

\[
COP = \frac{q_{\text{in}}}{W_{\text{cycle}}}
\]  

(22)

Thus, for the cycle setup shown in Fig. ?? with three identical evaporators:

\[
COP = \frac{3q_e}{q_a + W_m}
\]  

(23)

3.2.2. Critical Heat Flux

In the present refrigeration system for high flux removal applications, knowledge of the CHF condition is vital for safe system operation. When the imposed heat flux exceeds the CHF, the high heat flux together with the sudden decrease in heat transfer coefficient will cause the temperature of the heat dissipating device to rise high enough and eventually destroy the device.

The Katto correlation [34] is a well established correlation for CHF prediction in tubular channels, and it is expressed as:

\[
q_{\text{CHF}} = q_{\text{co}}\left(1 + \frac{K\Delta h_i}{h_{fg}}\right)
\]  

(24)

where \( q_{\text{co}} \) is the basic CHF, \( K \) is the inlet subcooling parameter, \( \Delta h_i \) is the inlet subcooling enthalpy, and \( h_{fg} \) is the latent heat of evaporation. Detailed interpretation of these terms can be found in Katto’s original paper [34], and the CHF generally has an increasing trend with increasing mass flow rate and inlet subcooling.
3.3. Parametric Study

To gain more insight into the present refrigeration system for high heat flux removal, a parametric study is conducted to study the effect of second fluid temperature \( T_{\text{cool}} \), evaporator heat load \( q_e \), compressor speed \( \omega \), EEV percentage opening \( A_v \), and accumulator heat \( q_a \) on system performance.

3.3.1. Second Fluid Temperature \( T_{\text{cool}} \)

In the simulation setup, cooling water is chosen to be the second fluid of the condenser to which the heat load is rejected. It is assumed that its temperature \( T_{\text{cool}} \) can vary from 25\(^\circ\)C to 35\(^\circ\)C, reflecting the possible weather and seasonal changes. Fig. 2 shows the cycle \( p - h \) plot for \( T_{\text{cool}} = 25\(^\circ\)C, 30\(^\circ\)C, and 35\(^\circ\)C, respectively, with \( q_e = 2500W \), \( q_a = 2000W \), \( \omega = 3000rpm \), and \( A_v = 20\% \).

It is shown in Fig. 2 that as the cooling water temperature increases, the cycle steady states shift upward on the \( p - h \) plot, indicating an increase in the whole system pressure level and, hence, increased evaporation and condensation temperature. The increased condenser temperature is necessary to maintain a sufficient temperature difference between the refrigerant and cooling water to reject the heat load. Also revealed in Fig. 2 is the larger pressure difference across the compressor as the \( T_{\text{cool}} \) increases, which leads to more compressor power and hence a decreasing system COP. It is observed in Fig. 2 that the refrigerant quality in the evaporator increases with \( T_{\text{cool}} \) and thus results in lower CHF.

3.3.2. Evaporator Heat Load \( q_e \)

Although specifically designed for high heat flux removal, the present refrigeration system is also expected to handle the low heat loads and corresponding low heat fluxes when the heat generating device is at standby mode. Fig. 3 shows
the cycle steady states on the \( p - h \) plot for \( q_e = 500W, \) 1500W, and 2500W, respectively, with \( q_a = 2000W, \) \( \omega = 3000rpm, \) \( A_v = 20\%, \) and \( T_{cool} = 30^\circ C. \)

It can be observed from Fig. 3 that a higher heat load will bring an increase in the pressure level of the evaporator and, hence, higher evaporation temperature. As the heat load increases, the exit quality of the evaporator also increases correspondingly. The wide range of heat loads from 500W to 2500W are all successfully removed without triggering the CHF condition, but higher system COPs are achieved for higher heat loads, implying that the present refrigeration system is more efficient in high heat flux removal.

3.3.3. Compressor Speed \( \omega \)

Compressor power is a major part of the power consumption of the refrigeration cycle for high heat flux removal, which directly affects the system COP. Fig. 4 shows the cycle steady states on the \( p - h \) plot for \( \omega = 2000rpm, \) 3000rpm, and 4000rpm, respectively, with \( q_e = 2500W, \) \( q_a = 2000W, \) \( A_v = 20\%, \) and \( T_{cool} = 30^\circ C. \)

With increasing compressor speed \( \omega, \) the cycle mass flow rate \( \dot{m} \) increases accordingly and, thus, lower evaporation temperatures and higher CHFs are achieved as shown in Fig. 4. On the other hand, compressor power \( W_m \) increases with the compressor speed \( \omega \) and results in decreased COP.

3.3.4. EEV Percentage Opening \( A_v \)

Fig. 5 shows the cycle steady states on the \( p - h \) plot for \( A_v = 12\%, \) 20\%, and 28\%, respectively, with \( q_e = 2500W, \) \( q_a = 2000W, \) \( \omega = 3000rpm, \) and \( T_{cool} = 30^\circ C. \)

It can be observed from Fig. 5 that the pressure difference between the condensation side and the evaporation side of the cycle experiences a bigger change when
$A_v$ is changed from 20% to 12%, compared with the case when $A_v$ is changed from 20% to 28%. This means that the cycle steady state is more sensitive to the valve opening changes in the lower range. This observation agrees with equation 6, which states that the pressure drop across the valve $\Delta P \propto 1/A_v^2$. Another important trend in Fig. 5 is that a smaller valve opening helps reduce the evaporator inlet quality and, thus, increase the CHF as shown in Fig. 5. The larger pressure drop resulted from the smaller valve opening $A_v$, however, will require the compressor to do more work and, hence, reduce the system COP. The conflicting trend in CHF and COP as $A_v$ varies suggests that a trade-off has to be made between these two objectives in the steady-state system operation.

3.3.5. Heat Supplied to the Accumulator $q_a$

Based on the accumulator equations 7, 8, and 9, heat supplied to the accumulator, $q_a$, affects the exit quality of the evaporator at steady state. Higher $q_a$ will naturally decrease the system COP, but it also lowers the evaporator exit quality. With a given heat load, the evaporator inlet quality will drop accordingly and a higher CHF can be achieved in the evaporator since CHF increases with inlet subcooling.

Fig. 6 shows the cycle steady states on the $p-h$ plot for $q_a = 1000W$, $2000W$, and $3000W$, respectively, with $q_e = 2500W$, $\omega = 3000rpm$, $A_v = 20\%$, and $T_{cool} = 30^\circ C$. It can be seen from Fig. 6 that as $q_a$ increases, the refrigerant qualities at both the inlet and outlet of the evaporator decrease, which lead to increased CHF. The system COP, however, will decrease because of the higher power consumption of the cycle brought by larger $q_a$. Once again, we observe the necessary trade-off between the conflicting objectives of COP and CHF.
4. INVESTIGATION OF STEADY-STATE OPERATION OPTIMIZATION

4.1. Problem Formulation

The present refrigeration system is designed for the efficient and safe removal of high heat fluxes, thus the system COP has to be maximized while avoiding the CHF condition. Although it is a natural thought to post the CHF condition as a constraint in the system operation optimization, we must acknowledge that all the CHF correlations available are only accurate to a certain extent, and some safety margin has to be reserved to prevent the device burnout. With this concern in mind, the CHF condition can be treated as an additional objective to be maximized along with the system COP, thus giving the system designer the freedom to choose the CHF safety margin.

Of the external system inputs discussed earlier, the second fluid temperature \( T_{cool} \) is usually determined by the environment and not directly controllable. The steady-state optimization of the refrigeration system with given heat load \( q_e \) and second fluid temperature \( T_{cool} \) can thus be formulated as the following the bi-objective constrained optimization problem:

\[
\begin{align*}
\text{maximize} & \quad \{ \text{COP} \} \\
\text{subject to} : & \quad \omega_{\text{min}} \leq \omega \leq \omega_{\text{max}} \\
& \quad 0 \leq A_v \leq 100 \\
& \quad 0 \leq q_a \leq q_{a,\text{max}} \\
& \quad T_{\text{min}} \leq T_{\text{wall}} \leq T_{\text{max}}
\end{align*}
\]

The physical limitations of the compressor, EEV, and the accumulator embedded heater are specified in the first three constraints. In the fourth constraint, the
evaporator wall temperature $T_{\text{wall}}$ (representing the heat generating device temperature) is maintained below $T_{\text{max}}$ to avoid device burnout, and at the same time above $T_{\text{min}}$ to prevent moisture condensation on the device surface that may cause an electrical short or other adverse effects. $T_{\text{min}}$ can be set to be well above the room dew point temperature to leave some safety margin. This approach is adopted in IBM’s eServer z990 [26] to replace the airtight metal enclosure used in IBM’s S/390 server [23], which lowers costs and system complexity.

4.2. Optimization Approach and Results

For multiobjective optimization (MO) problems, Pareto solution is a well accepted approach. A Pareto solution is one for which any improvement in one objective can only take place if at least one other objective worsens. The Pareto frontier of a MO problem consists of the Pareto solutions, and it gives the system designer the information needed to make a trade-off between the conflicting objectives according to varying conditions and requirements. Among the many methods to obtain the Pareto frontier of a MO problem, the normalized normal constraint method [35] is used for the present bi-objective (COP and CHF) optimization problem because its performance is entirely independent of the design objective scales.

For the simulation setup shown in Fig. 7 with three identical evaporators of the same heat load and valve opening, the Pareto frontiers of the [COP, CHF] optimization with $q_e = 2500W$ ($q''_{\text{imposed}} = 156.6kW/m^2$) and $q_e = 1500W$ ($q''_{\text{imposed}} = 94.0kW/m^2$) are shown in Fig. 7. The optimization constraints are $2000\text{rpm} \leq \omega \leq 5000\text{rpm}$, $0 \leq A_v \leq 100$, $0W \leq q_a \leq 5000W$, $25^\circ C \leq T_{\text{wall}} \leq 60^\circ C$.

Each point along the Pareto frontiers in Fig. 7 corresponds to the [COP, CHF]
of a Pareto solution obtained by optimizing the system input \((q_a, \omega, A_v)\) for a specific heat load \(q_e\), and the conflict between COP and CHF can be easily observed since the CHF decreases as system COP increases. To prevent the device burnout, solutions along the Pareto frontier with CHF higher than the imposed heat flux \(q''_{\text{imposed}}\) can be chosen for the steady-state operation. Although it is desirable to select an operating point with a higher CHF and, hence, a larger safety margin, it is observed in Fig. 7 that such an operating point will have a lower COP. The system designer thus has to make a trade-off between the system COP and CHF.

Fig. 7 reveals that the refrigeration system is more efficient in handling higher heat fluxes. The highest system COPs achieved without violating the CHF condition are around 2.5 for \(q_e = 2500\, \text{W}\) and 1.8 for \(q_e = 1500\, \text{W}\), respectively. The reason can be attributed to the fact that a heated accumulator is used in the system to fully vaporize the two-phase flow leaving the evaporator, and it can be found from Eqn. 23 that for the same system power input \((q_a + W_m)\) a larger heat load \(q_e\) results in a higher system COP. Another important fact shown in Fig. 7 is the slower slope of the Pareto frontier of \(q_e = 2500\, \text{W}\) compared with that of \(q_e = 1500\, \text{W}\), indicating that the system COP can be improved without compromising CHF a lot when handling higher heat fluxes. In Fig. 7 both Pareto frontiers have decreasing slope with increasing system COP, a feature amenable to the steady-state system operation optimization since the system suffers less CHF degradation as the system COP increases in its higher range.

Table 2 and 3 list the optimized system operation conditions \((q_a, \omega, A_v)\) and main system performance indices for the two Pareto frontiers shown in in Fig. 7, in which \(T_{\text{wall},i}\) and \(T_{\text{wall},o}\) are the evaporator wall temperatures at the inlet and outlet. Because of the increasing refrigerant quality and associated decreasing heat
transfer coefficient along the flow direction of the evaporator, $T_{wall,i}$ and $T_{wall,o}$ are the lowest and highest evaporator wall temperatures, respectively, when a uniform heat flux is applied. The system trends revealed in the parametric study of section 3.3 are also manifested in the Pareto optimization results shown in Table 2 and 3 for $q_e = 2500\, W$ and $q_e = 1500\, W$, respectively. As $q_a$ decreases, the system COP increases because of less energy consumption, and at the same time the evaporator exit quality increases which leads to lower CHF. For both Pareto frontiers, higher CHFs are achieved with smaller EEV opening and higher heat input supplied to the accumulator. It is also noted that all the Pareto solutions in Table 2 and 3 have the compressor speed at the lower bound of the optimization constraint. One possible explanation is that for the two heat loads considered the compressor running at this speed is capable of delivering sufficient refrigerant mass flow rate, but is less energy efficient than the accumulator heater in regulating the CHF, and thus the compressor is maintained at this low speed to achieve better system COP.

Although the present study is specifically targeted at high heat flux removal and the optimization results above show that higher efficiency is attained with higher heat flux, the possibility of removing low heat fluxes using the same system structure is also investigated. It is found that for much lower heat loads and heat fluxes ($q_e = 500\, W$ and $q''_{\text{imposed}} = 31.3\, kW/m^2$, for example), the main challenge is to maintain the evaporator wall temperature high enough (not lower than $25^\circ C$ in the optimization) to avoid ambient moisture condensation. To achieve this goal, either the accumulator heat input $q_a$ can be increased or the system mass flow rate can be decreased, and apparently the latter option is more energy efficient. This implies that either a compressor capable of delivering a wide range of mass flow rate is need, or parallel compressors of varying capacities can be used to cover the
wide range of mass flow rate needed.

5. EXPERIMENTAL SETUP AND MODEL VALIDATION

The experimental testbed built in Rensselaer Polytechnic Institute for high heat flux removal investigation is shown in Fig. 8. In the three evaporators, the imposed heat fluxes come from the cartridge heaters immersed in the refrigerant and each cartridge heater can provide up to $2.5\, kW$ heat output. Using the EEVs upstream of each evaporator, the system can be reconfigured to operate with one, two, or three evaporators simultaneously. The testbed is equipped with two compressors (small and medium) to allow for a wide range of operating conditions and is also fully instrumented with temperature, pressure, and mass flow rate sensors.

While the parameter identification of system components (compressors and EEVs) is still underway, preliminarily model validation against the testbed has been undertaken using the initial parameter identification results. Fig. 9 shows the cycle steady-state comparison between the experiments and model prediction. In case 1, the testbed operates with only one evaporator and the small compressor, while in case 2 two identical evaporators and the medium compressor are in operation.

It can be seen from the $p−h$ plots that the modeling prediction matches the experimental data well in both cases, with only noticeable differences at the exits of the accumulator and compressor. The discrepancy at the accumulator and compressor exits can be attributed to the efficiency correlations used in the compressor model and the 20 inches of piping connecting the accumulator and the compressor. It is found through modeling that the compressor exit temperature is sensitive to the isentropic efficiency and hence an accurate correlation is desired. Also found
through experiments is the fact that although the piping connecting the accumulator and the compressor is insulated, some heat can still be gained from the ambient and thus slightly superheat the exit of the accumulator, which is assumed to be saturated vapor in the steady-state modeling. To solve this problem, more accurate correlations are being developed to better model the compressor and pipe model will be included in future modeling work to capture the heat transfer process and pressure change in the piping.

6. CONCLUSIONS AND FUTURE WORK

In this paper, the steady-state modeling and operation optimization of a refrigeration system for high heat flux removal is presented. To deal with the CHF condition in high heat flux removal, the evaporator is designed to operate with two-phase flow only, and thus an liquid accumulator with embedded heater is included in the cycle for the safe operation of the compressor and cycle active charge regulation. Using the steady-state model, the effect of various external system inputs on the system performance is investigated, and Pareto optimization is applied to optimize the system steady-state operation for given heat loads. The initial experimental data from the testbed show good prediction ability of the model, and validation of the steady-state system operation optimization approach is also underway.

7. ACKNOWLEDGEMENTS

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References


[19] D. Richardson, H. Jiang, D. Lindsay, and R. Radermacher. Optimization


**TABLE CAPTIONS**

**Table 1.** Comparison of vapor compression cycle simulation tools

**Table 2.** Pareto solutions for $q_e = 2500W$ ($q_{imposed}'' = 156.6kW/m^2$)

**Table 3.** Pareto solutions for $q_e = 1500W$ ($q_{imposed}'' = 94.0kW/m^2$)
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FIGURE CAPTIONS

Fig.1. Refrigeration system structure for high flux removal.

Fig.2. Effect of $T_{cool}$ on system performance (P-h plot).

Fig.3. Effect of $q_e$ on system performance (P-h plot).

Fig.4. Effect of $\omega$ on system performance (P-h plot).

Fig.5. Effect of $A_v$ on system performance (P-h plot).

Fig.6. Effect of $q_a$ on system performance (P-h plot).

Fig.7. Pareto frontier for $[COP \ CHF]^T$ optimization.

Fig.8. Experimental testbed.

Fig.9. Comparison between experimental data and modeling prediction
(case 1: $q_e = 1200W$, $A_v = 6\%$, $T_{cool} = 25.0^\circ C$, $\omega = 4050rpm$,
case 2: $q_{e1} = q_{e2} = 1500W$, $A_{v1} = A_{v2} = 35\%$, $T_{cool} = 26.7^\circ C$, $\omega = 4200rpm$).

36
Fig. 1
Fig. 2

- Saturated vapor
- Saturated liquid

- $T_{\text{cool}} = 25^\circ\text{C}$, COP=2.39, CHF=171 kW/m$^2$
- $T_{\text{cool}} = 30^\circ\text{C}$, COP=2.25, CHF=162 kW/m$^2$
- $T_{\text{cool}} = 35^\circ\text{C}$, COP=2.13, CHF=151 kW/m$^2$
Fig. 3
Fig. 4

- **saturated vapor**
- **saturated liquid**

- $\omega=2000\text{rpm}, \text{COP}=2.70, \text{CHF}=258\text{kW/m}^2$
- $\omega=3000\text{rpm}, \text{COP}=2.25, \text{CHF}=273\text{kW/m}^2$
- $\omega=4000\text{rpm}, \text{COP}=2.01, \text{CHF}=283\text{kW/m}^2$

![Pressure vs. Enthalpy Diagram](image-url)
Fig. 5
Fig. 6

- **saturated vapor**
- **saturated liquid**

- $q_a = 1000\text{W}$, COP = 3.23, CHF = 151$\text{kJ/m}^2\text{h}$
- $q_a = 2000\text{W}$, COP = 2.25, CHF = 162$\text{kJ/m}^2\text{h}$
- $q_a = 3000\text{W}$, COP = 1.73, CHF = 171$\text{kJ/m}^2\text{h}$
Fig. 7
Fig. 8
Fig. 9